PROBLEMS FOR SOLUTION.

ALGEBRA.

337. Proposed by I. M. CURTISS, Brooklyn, N. Y.

Three regiments move north as follows: B is 20 miles east of A; C is 20 miles south of B, and each marches 20 miles between the hours of 5 a. m. and 3 p. m. A horseman with a message from C starts at 5 a. m. and rides north till he overtakes B, then sets a straight course for the point at which he calculates to overtake A, then sets a straight course for the next point at which he will again overtake B, then rides south to the point where he first overtook B, reaching that point at the same time as C, namely 3 p. m. What uniform rate of travel enabled the messenger to do this?

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let x=time required to overtake B. He travels $20+2x$ miles. Hence $\frac{20+2x}{x}$=his rate. Let y=time to go from B to A. He travels $2\sqrt{(100+y^2)}$ miles. $\frac{2\sqrt{(100+y^2)}}{y}$=his rate. After reaching B a second time he has left $10-x-2y$ hours to go $2x+4y$ miles.

$\therefore\ \frac{2x+4y}{10-x-2y}$=his rate. But his rate is uniform. Hence we get

$$\frac{20+2x}{x}=\frac{2\sqrt{(100+y^2)}}{y},\text{ or } 5x^2=5y^2+xy\ldots(1).$$

$$\frac{20+2x}{x}=\frac{2x+4y}{10-x-2y},\text{ or } x^2+2xy=50-10y\ldots(2).$$

If $x=vy$ in (1), we get $5v^2-v=5$ or $v=1.10499$.

$\therefore x=1.10499y$. This in (2) gives $14.41996y^2+10y=50$ or $y=1.54737$. $x=1.70983$. Rate$=20/x+2=13.69707$ miles an hour.

Also solved by V. M. Spunar. A. H. Holmes, and J. Scheffer.

338. Proposed by R. D. CARMICHAEL, Princeton University.

Prove that $\pi=3+\frac{1}{3}\cdot\frac{1}{1.2}-\frac{1}{5}\cdot\frac{1}{2.3}+\frac{1}{7}\cdot\frac{1}{3.4}-\frac{1}{9}\cdot\frac{1}{4.5}+\ldots$

Solution by S. LEFSEHETZ, East Pittsburg, Pa.

Let $S(x)=\sum_1^\infty(-1)^{n+1}\frac{x^{2n}}{n(n+1)(2n+1)}=\sum_1^\infty\left[\frac{1}{n}+\frac{1}{n+1}-\frac{4}{2n+1}\right](-1)^{n+1}x^{2n}$

$$=\sum_1^\infty(-1)^{n+1}\frac{x^{2n}}{n}+\sum_1^\infty(-1)^{n+1}\frac{x^{2n}}{n+1}+4\sum_1^\infty(-1)^2\frac{x^{2n}}{2n+1}$$